

A Dodecalogue of Basic Didactics from Applications of Abstract Differential Geometry to Quantum Gravity

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Abstract We summarize the twelve most important in our view novel concepts that have arisen, based on results that have been obtained, from various applications of Abstract Differential Geometry (ADG) to Quantum Gravity (QG). The present document may be used as a concise, yet informal, discursive and peripatetic conceptual guide-*cum*-terminological glossary to the voluminous technical research literature on the subject. In a bonus section at the end, we dwell on the significance of introducing new conceptual terminology in future QG research by means of ‘poetic language’.

Keywords Abstract differential geometry · Quantum gravity · Poetry

1 A Dodecalogue of ADG-Didactics on QG

Anastasios Mallios’ Abstract Differential Geometry (ADG) has been with us, at least in research monograph form, for almost a decade now [14]. During this time, it has enjoyed numerous applications to quantum gauge theories and gravity [15–17, 20–31, 38–41],¹ culminating recently in another two-volume up to date treatise [18, 19].

Below, an informal *résumé* of twelve novel concepts and main results that have been gathered from applying ADG to QG is given, a document that can at least provide the reader with

¹In the present text little reference to the ‘satellite’ (*i.e.*, non-ADG based) QG literature, such as Loop Quantum Gravity (LQG) or Causal Set (:Causet) Theory, is given. The reader should refer to these papers for extensive citations of research monographs and papers on QG. In [28, 40] especially, the reader can find fairly comprehensive and updated reference lists.

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a handy guide, as well as a sketchy glossary of pivotal new terms, to the existing literature ‘zoo’ on the subject. The list of a dozen items that follows is a distillation and summary of little, albeit important, lessons that this author has learned from employing, predominantly in collaboration with Mallios, ADG-theoretic concepts, techniques and results to QG and quantum gauge theories mainly in a finitistic-algebraic and categorical setting [25–28, 37–41].² Some of the twelve ‘lessonets’ (‘ $\mu\alpha\theta\eta\mu\alpha\tau\iota\kappa\acute{\alpha}$ ’=Greek for ‘*mathematics*’!)³ listed below are closely related to each other.

1. **Background spacetime manifoldlessness.** ADG-gravity⁴ is manifestly background spacetime manifold independent. There is no smooth locally Euclidean base space involved in formulating the (vacuum) Einstein equations, hence no space(time) interpretation of that base either.

2. **Half-Order Formalism.** The sole dynamical variable in ADG-gravity is an algebraic \mathbf{A} -connection field \mathcal{D}^5 acting (on the local sections of) a vector sheaf \mathcal{E} defined on an in principle arbitrary topological space X . The physical kinematical configuration space in the theory is the moduli space $\mathbf{A}_{\mathbf{A}}(\mathcal{E})/\text{Aut}\mathcal{E}$ of the affine space of connections \mathbf{A} modulo the (local) gauge transformations in the principal sheaf $\text{Aut}\mathcal{E}$. The ADG-formalism on gravity is called half-order formalism in order to distinguish it from the first-order one of Palatini–Ashtekar, and from the original second-order one of Einstein, both of which depend on a background smooth manifold for their differential geometric expression. From the ADG-theoretic vantage, gravity is regarded as a pure gauge theory since only the connection, and not the smooth metric (or equivalently, the smooth tetrad field), is a dynamical variable. It follows that the connection \mathcal{D} represents the gravito-inertial field and, unlike the $g_{\mu\nu}$ of GR, *not* the chrono-geometrical structure. There is no ‘spacetime geometry’ in ADG-gravity, or rather more mildly, if there is any ‘space’ (‘geometry’) at all, it is already encoded in the \mathbf{A} chosen.⁶ ‘Geometry’ (or indeed, ‘spacetime’) is completely encoded in our (generalized) measurements in \mathbf{A} . There is no geometry without measurement, without the production (‘recording’) of numbers of some sort.⁷ At the same time, (the products of) measurements (‘numbers’) are our own actions (and numbers our own artifacts/inventions), hence no physical reality, and no interpretation as the gravitational field living ‘out there’, should be given to the spacetime metric, like in the original formulation of GR. This is consistent with our viewing gravity as a pure gauge theory—i.e., that the gravitational field is simply the connection \mathcal{D} .

²For another fairly recent review on the subject, see [40]. The ‘*Glossary for ADG-Gravity*’ appended at the end of [28] may also be of help, although some of the terms and definitions found there are presently (*i.e.*, two years later!) slightly more refined, expanded and supported by further work that has been done in the meantime.

³In Greek, the word ‘ $\mu\acute{\alpha}\theta\eta\mu\alpha$ ’ means ‘*lesson*’; hence, ‘*mathematics*’ is used above as the diminutive of ‘*lessons*’ (‘ $\mu\alpha\theta\acute{\eta}\mu\alpha\tau\alpha$ ’).

⁴Following [27, 28, 38–41], we subsume the ADG-theoretic formulation of gravity (classical and/or quantum, with no significant distinction between these two epithets—see item 10 on the list) [17–19, 21, 22, 24–27, 29–31, 38–40] under the term ‘*ADG-gravity*’.

⁵Where \mathbf{A} is a sheaf of unital, commutative and associative \mathbb{K} -algebras ($\mathbb{K} = \mathbb{R}, \mathbb{C}$) called the *structure sheaf* of generalized arithmetics or coordinates. \mathbf{A} is chosen by the theorist (or the ‘*experimenter*’/‘*observer*’!) and it generalizes the usual structure sheaf \mathcal{C}_M^∞ of germs of smooth (usually taken to be \mathbb{R} -valued) coordinate functions on a differential manifold M .

⁶In the usual case $\mathbf{A} \equiv \mathcal{C}_M^\infty$, we have in mind here the notion of Gel’fand duality whereby the differential manifold M is the spectrum of the (non-normed) topological algebra sheaf \mathcal{C}_X^∞ .

⁷Normally, we assume that we measure real (\mathbb{R}) numbers.

3. **All differential geometry boils down to \mathbf{A} .** To be sure, there would be no *differential* geometry without a differential, in fact, without \mathcal{D} , and \mathcal{D} has \mathbf{A} as its domain of definition. There is an apparent oxymoron here: we are inclined to give physical reality (:independence from our observations/measurements in \mathbf{A}) to the gravitational connection field \mathcal{D} , yet here we seem to claim that \mathbf{A} is vital for its very definition. There is no paradox: the ADG-vacuum Einstein gravitational equations are expressions involving the *curvature* of \mathcal{D} , which is a ‘*geometrical object*’ (:an $\otimes_{\mathbf{A}}$ -tensor,⁸ or equivalently, an \mathbf{A} -sheaf morphism) in the theory, hence the dynamics is functorial relative to \mathbf{A} . On the other hand, \mathcal{D} is *not* an $\otimes_{\mathbf{A}}$ -tensor, hence it eludes our measurements in \mathbf{A} . \mathbf{A} is simply employed to effectuate (:geometrically represent) \mathcal{D} on a vector sheaf \mathcal{E} ,⁹ which is the carrier (:representation-action) space of \mathcal{D} .

4. **Gauge Theory of the Third Kind.** The half-order ADG-gravity, although by definition (and sheaf-theoretic construction!) a local gauge theory, it is not local in the usual sense of the current so-called gauge theories of the second kind since no background spacetime manifold is employed to localize the ADG-gravitational field and the symmetries of the dynamical equations that it obeys (:the vacuum Einstein equations). It follows that the group $\text{Diff}(M)$ of smooth external spacetime (as opposed to internal gauge) symmetries does not figure in the theory, and all the ‘symmetries’ of the vacuum gravitational dynamics in $\text{Aut}\mathcal{E}$ are ‘internal’ to the ADG-gravitational field $(\mathcal{E}, \mathcal{D})$.¹⁰

5. **Unconstrained Gauge Gravity.** From the above, it follows that ADG-gravity is an external smooth spacetime unconstrained pure gauge theory. The primary, externally imposed (by the theorist, or the ‘experimenter’) spacetime gauge constraints have been eliminated in ADG-gravity, so that what remains is the dynamically autonomous ADG-gravitational connection field \mathcal{D} .

6. **Algebraicity, Representation, Categoricity and Functoriality.** ADG-gravity is a purely algebraic (:sheaf-theoretic) scheme. Its main structural element, the algebraic \mathbf{A} -connection field \mathcal{D} is a linear Leibnizian sheaf morphism acting categorically on \mathcal{E} ’s local sections to change them dynamically. In turn, \mathcal{E} is regarded as the associated (:representation or carrier/action) sheaf of the principal group sheaf $\text{Aut}\mathcal{E}$, while from a geometric (pre)quantization and second quantization vantage, its local sections represent local quantum-particle states of the ADG-fields involved. The ADG-field—by definition, the pair $(\mathcal{E}, \mathcal{D})$ —has thus been coined ‘particle-field pair’, and the vacuum Einstein equations $\mathcal{R}(\mathcal{D})[\mathcal{E}] = 0$ have a straightforward geometrical interpretation: the connection acts via its (Ricci) curvature on the local particle states (:local sections) of \mathcal{E} to change them dynamically. Moreover, since the said curvature is an \mathbf{A} -morphism (or equivalently, an $\otimes_{\mathbf{A}}$ -tensor, with $\otimes_{\mathbf{A}}$ the homological tensor product functor), the dynamics is functorial relative to our generalized coordinate measurements in (:coordinatizations of the connection field \mathcal{D} by) \mathbf{A} .¹¹ This \mathbf{A} -functoriality (or equivalently, the $\text{Aut}\mathcal{E}$ -invariance) of the vacuum Einstein dynamics is the ADG-analogue of the Principle of General Covariance (PGC) of the C^∞ -smooth manifold

⁸ $\otimes_{\mathbf{A}}$ is the homological tensor product functor between the (vector) sheaf categories involved in ADG-gravity.

⁹ By definition, \mathcal{E} is locally of the form \mathbf{A}^n (n a positive integer called the rank of the vector sheaf \mathcal{E}).

¹⁰ Of course, in case one chooses $\mathbf{A} \equiv C_X^\infty$ (i.e., $X \equiv M$ by Gel’fand duality), $\text{Aut}(M) := \text{Diff}(M)$ and one falls back to the usual scenario (:the manifold based GR).

¹¹ In our scheme, coordinatizing the algebraic connection field \mathcal{D} is tantamount to representing it on \mathcal{E} , which by definition is locally of the form \mathbf{A}^n (:a locally free \mathbf{A} -module of finite rank n). In turn, \mathcal{E} is the associated (:representation) sheaf of $\text{Aut}\mathcal{E}$.

based GR in which the PGC is effectuated via $\text{Diff}(M)$.¹² All in all, ADG is an entirely algebraic way of doing differential geometry whereby all that is of technical import and physical significance is the algebraic relations between the sections of the sheaves involved, without the mediation in the guise of (smooth) coordinates of a background geometrical differential (spacetime) manifold.

7. Third Quantization. Third quantization is ADG-field quantization. As noted above, the ADG-fields are defined to be pairs $\mathfrak{F} := (\mathcal{E}, \mathcal{D})$, and 3rd-quantization pertains to setting up non-trivial local commutation relations between certain characteristic local (:differential) forms that uniquely characterize sheaf cohomologically the vector sheaves \mathcal{E} and the connections \mathcal{D} on them. Based on the ADG-field semantics coming from applications of ADG to geometric (pre)quantization and 2nd-quantization, the said characteristic forms may be interpreted as abstract particle-position and field-momentum ‘determinations’ (:operations or actions), so that their commutation relations can be regarded as abstract canonical-type of local Heisenberg uncertainty relations. Moreover, since the ADG-formalism is inherently background spacetime manifoldless, 3rd-quantization of ADG-gravity, in contradistinction to the connection, but also manifold, based Loop Quantization in canonical QGR, it does not involve at all quantization of the spacetime continuum itself. Rather, it is an autonomous quantization scenario for the ADG-fields \mathfrak{F} ‘in-themselves’, as it involves solely their abstract particle-position (\mathcal{E}) and field-momentum (\mathcal{D}) parts. 3rd-quantization is manifestly \mathbf{A} -functorial, hence expressly fully (:generally) covariant, whereas it takes a lot of effort to secure that the usual canonical $\text{Diff}(M)$ -constrained Dirac quantization of GR be (generally) covariant (even in the Ashtekar variables formulation of GR, which significantly simplifies the primary spacetime-diffeomorphism constraints). Finally, ‘observables’ in 3rd-quantized ADG-vacuum Einstein gravity are all the \mathbf{A} -morphisms (*alias*, $\otimes_{\mathbf{A}}$ -functors like the curvature of \mathcal{D})—the geometrical (: \mathbf{A} -measurable) dynamical objects.

8. No Singularities and No Infinities’. It is plain that in a purely algebraico-categorical scheme such as ADG-gravity, there are no infinities, since the infinitary (:limiting) processes of the usual continuum mediated Analysis that we have hitherto used in QG research are replaced by Algebra (:the dynamical relations between the geometrical objects in the theory—the $\otimes_{\mathbf{A}}$ -tensors or \mathbf{A} -morphisms),¹³ and there are no (analytic) infinities in the latter. Furthermore, singularities of all sorts, even non-linear distributional ones, are seen to be absorbed (or integrated) in \mathbf{A} so that by virtue of the \mathbf{A} -functoriality of the ADG-gravitational dynamics, the latter is seen *not* to break down in any differential geometric sense in the vicinity of the singular *loci*. The ADG-gravitational field (\mathcal{E}, \mathcal{D}) (and the \mathbf{A} -functorial vacuum Einstein equations that it defines via its curvature \mathbf{A} -morphism) ‘sees’ through the singularities which are embodied in \mathbf{A} .

9. No Fundamental Scales, No Continuum/Discretum Dichotomy. Much in the same way that we cannot accept that there are singularities and infinities in Nature, or that there is a spacetime (continuum) separate from (i.e., externally prescribed to) the dynamical fields themselves—a continuum that is responsible for the singularities and their associated (analytic) infinities in the first place—we cannot accept that there are fundamental space-time scales above which the law of GR holds, but below which another set of laws (those of the QG we are after) does. Such regularization scales are normally posited in order to

¹²To be sure, since by definition \mathcal{E} is locally a finite power of \mathbf{A} , $\mathcal{E}|_U := \mathbf{A}^n(U)$, $\text{Aut}\mathcal{E}$ is also locally isomorphic to the general coordinate group sheaf: $\text{Aut}\mathcal{E}(U) = \mathcal{GL}(n, \mathbf{A}(U)) \cong M_n(\mathbf{A}(U))^\bullet$, ($U \subset X$ open).

¹³More generally, in ADG we have effectively replaced, in a *differential* geometric context, Analysis by sheaf cohomology (i.e., ‘space’ by homological algebra) in a way analogous to what Grothendieck did in *algebraic* geometry.

make sense out of the non-renormalizable infinities of our manifold based Analysis and have nothing to do with Nature (i.e., with the dynamical field laws themselves).

Mutatis mutandis for the continuum/discretum dichotomy in ADG-gravity: if one ‘concocts’ supposedly fundamental spacetime scales, such as Planck’s, out of the universal constants (e.g., \hbar , c , G) which in the theory are represented by (global) sections of the constant sheaf \mathbf{K} , the \mathbf{A} - and, plainly, the \mathbf{K} -functoriality¹⁴ of the ADG-gravitational dynamics indicates that the theory¹⁵ recognizes no such cut-off scales delimiting and separating (space-time) continuum from discretum based scenaria.¹⁶ Since no base spacetime is involved in ADG-gravity, it is meaningless to make such continuum/discretum distinctions either. It follows that top-down/bottom-up distinctions between various approaches to QG, such as LQG and Causet Theory respectively, lose their meaning in ADG-gravity.¹⁷ The field-law of gravity appears to be more unified (‘unitary’) in ADG-gravity than in other approaches to QG that a priori suppose a (background spacetime) continuum/discretum distinction. To stress it again, ADG-gravity has no background continuum or discretum spacetime commitments.

10. No Classical/Quantum Schism. Perhaps the most radical ‘no’ of the ADG-gravitational didactics is that renouncing the classical/quantum separation (‘Heisenberg’s *schnitt*’). Generally speaking, it seems intuitively naive and contrived to assume that there are classical physical systems and theories describing them, and moreover, that a formal procedure of quantization should take us to their fundamental quantum correspondents. Conversely, it seems artificial and presumptuous to demand that any quantum description of a physical situation should yield at an appropriate so-called correspondence limit a classical description (of the same system!), let alone to adopt as a criterion for the acceptability of a quantum theory whether it yields, in a suitable ‘limit’ sense, its classical counterpart as a coarse, effective theory. There is a psychological factor inherent in our ‘correspondence principle’ conservatism: we seem to trust more ‘classical’ than ‘quantum’ theories, because the former have been more experimentally vindicated (or at least they have been around longer than quantum ones, and they have been longer worked out mathematically and better understood conceptually). Accordingly, we demand from our quantum theories to set, via a suitable correspondence principle, one foot on the classical domain for stability and acceptability’s sake. By contrast, in the past we have argued extensively that our ADG-gravity is already (3rd) quantum since the ADG-gravitational field (\mathcal{E} , \mathcal{D}), and the law that it defines, has *ab initio* quantum traits inherent in its structure. Thus, the ADG-gravitational field is in no need of a (formal) process of quantization, while 3rd-quantization is a procedure of explicating its ‘self-quantum’ nature (not a quantization of some ‘classical’ sort of ADG-gravity, which does not exist from our perspective).¹⁸

11. ADG-Field Monadology, Unitarity, Quantum Dynamical Autonomy and Synvariance. From the ADG-perspective, the field $\mathfrak{F} := (\mathcal{E}, \mathcal{D})$ is, to paraphrase Einstein [4], “an independent, not further reducible fundamental concept”. In fact, it is an ‘entelechi’,

¹⁴It is tacitly assumed that $\mathbf{K} \xrightarrow{\subseteq} \mathbf{A}$.

¹⁵Implicit here is the ‘credo’ that a physical theory is nothing else but the dynamical principles and structures that it assumes, as well as the dynamical laws that it is able to formulate according to them.

¹⁶For example, the continuum based GR distinguished from the ‘discrete’ causal set (causet) theory.

¹⁷On the other hand, by ADG-means we are able pass through the horns of the continuum/discretum dilemma, bring together and fuse to a certain extent various central structures involved in the continuum based LQG and the discrete causet theory [25–28, 36, 38, 39].

¹⁸To be sure, as a formal ‘correspondence principle’ in ADG-gravity one could take the assumption $\mathbf{A} \equiv C_M^\infty$, but this should not be regarded as a transition from quantum to classical ADG-gravity: in 3rd-quantum ADG-gravity all structure sheaves are on a par.

dynamically autonomous quantum structure, since on the one hand the dynamical equations (Einstein's equations) are defined solely in terms of it, while on the other, the 3rd-quantum Heisenberg uncertainty relations mentioned above are defined intrinsically—i.e., only in terms of its two constituent parts: \mathcal{E} and \mathcal{D} .¹⁹ As mentioned before, since no spacetime (continuous or discrete) external to the ADG-gravitational field is posited, the $\text{Diff}(M)$ -modeled PGC of GR is hereby replaced by $\text{Aut}_{\mathbf{A}}\mathcal{E}$ so that 'covariance' is replaced by 'synvariance': the ADG-gravitational field varies dynamically 'in-itself', not relative to an external ('parameter') space(time). In this respect it reminds one of the Leibnizian monads [13], which are entelechic, windowless entities.; although, the ADG-fields are relational (:algebraic) entities that can interact with each other. Moreover, this external spacetimelessness makes the ADG-gravitational field an unconstrained, pure gauge quantum system (gauge field theory of the 3rd-kind; 3rd-quantization).

Finally, in the context of ADG, the epithet 'unitary' to 'field theory' pertains less to Einstein's original vision of 'one single field encompassing all forces'²⁰ and more to (also Einstein's, less popular however, vision of) a 'total field' obeying (defining) total dynamical (differential) equations that are 'free' from (i.e., in no way impeded by, let alone breaking down in the presence of) singularities, it incorporates its particle-quanta as singularities in the law (differential equation) that it defines, and (as a bonus to Einstein's vision—and something that Einstein could not have possibly envisioned in the manifold and *in extenso* CDG-based field theory that he was espousing) the background spacetime continuum plays no role whatsoever in the field's autonomous dynamics ('autodynamics'). *In summa*, a genuinely unitary field theory in our ADG-theoretic sense is concerned only with the field, the whole field and nothing but the field.

12. The Principles of Relativity of Differentiability, ADG-Field Realism and Field-Solipsism. Since the basic tenet of ADG is that all differential geometry boils down to (our choice of) \mathbf{A} , the Principle of Algebraic Relativity of Differentiability (PARD) pertains to different choices (on the part of the, external to the ADG-gravitational \mathbf{A} -connection field \mathcal{D} , theorist/observer/experimenter) of structure sheaf \mathbf{A} of generalized arithmetics (:coordinates or measurements), hence of different algebras of differentiable functions and different representation vector sheaves carrying the connection field \mathcal{D} , while at the same time leaving the dynamical equations form-invariant.²¹ Categorically, PARD is modeled after natural transformation-type of changes, which leave 'invariant' the \mathbf{A} -functorial vacuum ADG-gravitational dynamics.

A corollary of PARD is the Principle of ADG-Field Realism (PFR): one is in principle free to choose (according to the physical situation/problem one encounters) different \mathbf{A} s to 'measure' and geometrically represent (on $\mathcal{E} \stackrel{\text{loc}}{=} \mathbf{A}^n$), but the field and the law that it obeys (better, defines) remain unaffected by such changes. The field exists 'out there', independently of our 'measurements' (of it) in \mathbf{A} and, accordingly, of our geometrical representations in \mathcal{E} .

This ADG-field realism, together with the aforesaid quantum dynamical ADG-field autonomy, make us coin the philosophy underlying ADG-field theory 'ADG-field solipsism'. For, solipsism is 'pure realism'.

¹⁹For this, the field $\mathfrak{F} := (\mathcal{E}, \mathcal{D})$ has been coined 'self-quantum'.

²⁰Indeed, currently 'unitary' has degenerated to 'unified' in the sense above: one field for all forces.

²¹For example, such changes (in our choice) of \mathbf{A} may be made, because we might wish to include an occurring singularity in a new functional structure sheaf that the old one did not embody.

2 Poetry in Motion and in Action: the Future of QG Research

Descending to the Quantum Deep: the ‘Experience-to-Theoretical Physics-to-Mathematics-to-Philosophy-to-Poetry’ Ascension In QG research, because of the glaring absence of experimental data (in fact, of any controlled laboratory experiments!)²² to verify (or more importantly, to falsify!) our theories, the theoretical/mathematical physicist finds herself in the fortuitous position of being free to roam in unconstrained, uninhibited theory making, with sole guiding tools ‘aesthetic’ elements such as conceptual simplicity, economy and beauty, backed by mathematical abstraction, generality, rigor and logical consistency. This has been appreciated as early as Dirac’s days [3], who implored theoretical physicists to explore and use all the *mathematical* resources at their disposal, and divesting for a while experiments of their theory checking and guiding role.

Faddeev, for example, maintained recently [6] that we should finally break away from the classical theory-developmental route followed so far by theoretical physics, which can be schematically represented by the cycle:

experiments → predictions → mathematical elaborations/formulations
→ further experiments

and instead implore all our mathematical resources to plough deeper into the foundations of ‘physical reality’, leaving experiments (and experimentalists!) to ‘catch up’ with the new mathematics (and with theoreticians!), not the other way round. In this regard, we would like to borrow from [6] some telling remarks made by Dirac from the aforementioned paper [3]:²³

“... The steady progress of physics requires for its theoretical foundation a mathematics that gets continually more advanced. This is only natural and to be expected. What, however, was not expected by the scientific workers of the last century was the particular form that the line of advancement of the mathematics would take, namely, it was expected that the mathematics would get more complicated, but would rest on a permanent basis of axioms and definitions, *while actually the modern physical developments have required a mathematics that continually shifts its foundation and gets more abstract...* It seems likely that this process of increasing abstraction will continue in the future and that advance in physics is to be associated with a continual modification and generalization of the axioms at the base of mathematics rather than with logical development of any one mathematical scheme on a fixed foundation.²⁴ (I)

There are at present fundamental problems in theoretical physics awaiting solution [...] ²⁵ the solution of which problems will presumably require a more drastic revision of our fundamental concepts than any that have gone before. Quite likely these changes will be so great that it will be beyond the power of human intelligence to get the necessary new ideas by direct attempt to formulate the experimental data in mathematical terms. The theoretical worker in the future will therefore have to proceed in a more

²²Although we are passive receptors of cosmological data from the early universe.

²³The quotation below is split into two paragraphs (I and II), on which we comment separately after it.

²⁴Our emphasis.

²⁵Dirac here mentions a couple of outstanding mathematical physics problems of his times. We have omitted them.

indirect way. *The most powerful method of advance that can be suggested at present is to employ all the resources of pure mathematics in attempts to perfect and generalize the mathematical formalism that forms the existing basis of theoretical physics, and after²⁶ each success in this direction, to try to interpret the new mathematical features in terms of physical entities²⁷... (II)*"

- **(I)** The words from this paragraph to be highlighted with ADG-gravity in mind are: ‘*a mathematics that gets more abstract*’ and ‘*advance in physics is to be associated with a continual process of abstraction [leading to a] modification and generalization of the axioms at the base of mathematics*’. Indeed, the axiomatic ADG essentially involves an abstraction of the fundamental notions of modern differential geometry (e.g., connection), resulting in an entirely algebraic (:sheaf-theoretic) modification and generalization of the latter’s basic axioms [14, 15, 18, 19]. And it is precisely this abstract and generalized character of ADG that makes us hope that its application could advance significantly (theoretical) physics, and in particular, QG research. For, to quote again Einstein, in the quantum deep we must look for “*a purely algebraic method for the description of reality*” [4].²⁸
- **(II)** In this paragraph, apart from breaking from the traditional cycle ‘experiment-theory-more experiment’ mentioned above (i.e., Dirac’s anticipation that ‘*new ideas [won’t come] by direct attempts to formulate the experimental data in mathematical terms*’), what should be highlighted is on the one hand Dirac’s prompting us ‘*to generalize the mathematical formalism that forms the existing basis of theoretical physics*’, and on the other, ‘*to try to interpret the new mathematical features in terms of physical entities*’. Again, ADG comes to fulfill Dirac’s vision, since *the* (or at least the bigger part of the) mathematics that lies at the heart of current theoretical physics—namely, (the formalism of) *differential geometry* (i.e., CDG on smooth manifolds)—is abstracted and generalized, while *after* this generalization has been achieved, the physical application and interpretation (of ADG’s novel concepts and features) has been carried out, especially in the theoretical physics’ field of quantum gauge theories and gravity research. We believe that this is ‘*a powerful method of advance*’ indeed.

However, this too is not enough. Existing mathematical concepts, structures and techniques also come hand in hand with implicit assumptions, hidden preconceptions and prejudices associated with their historical development, i.e., with past problems other than QG(!) that they were invented in order to formulate, tackle and (re)solve. Such preconceptions are very hard to forget at the primary stages of theory making, let alone to shed them altogether, especially when they have proved to be experimentally successful in the past. Again Einstein, for example, has given us a warning call regarding our almost religious abiding by old, tried-and-tested concepts [5]:

“... Concepts which have proved useful for ordering things easily assume so great an authority over us, that we forget their terrestrial origin and accept them as unalterable facts. They then become labeled as ‘conceptual necessities’, ‘a priori situations’, etc.”²⁹

²⁶Dirac’s own emphasis.

²⁷Again, our emphasis throughout.

²⁸Alas, for Einstein, the continuum spacetime and *in extenso* CDG-based field theory was simply incompatible with the finitistic-algebraic quantum theory [46], a divide that ADG has come a long way to finally bridge [25–28, 38–41].

²⁹Think for instance of the apparently fundamental notion of the ‘*spacetime continuum*’: “*time and space are modes by which we think, not conditions in which we live*” (as quoted by Manin in [32]).

The road of scientific progress is frequently blocked for long periods by such errors. It is therefore not just an idle game to exercise our ability to analyze familiar concepts, and to demonstrate the conditions on which their justification and usefulness depend, and the way in which these developed, little by little... ”

For this, few people have suggested to go even a bit further, past mathematics, and into the realm of *philosophy* to look for novel QG research resources. ’t Hooft, for example [48], insists that:

“... The problems of quantum gravity are much more than purely technical ones. They touch upon very essential philosophical issues... ”

For us, this will not suffice either. Philosophy too comes burdened with a host of a priori concepts and assumptions.³⁰ Paraphrasing Finkelstein [9], “*in the quantum deep one must travel light*”. Alas, perhaps because of a deep psychological tendency towards security (and an instinctive, biological one, for survival [50]), we tend to abide by what we already know and (think) we understand (or believe to have a firm hold of backed by numerous practical applications), and we take few ‘conservative risks’ (pun intended) towards standing bare, ignorant (but, exactly thanks to this ignorance, uninhibited and unbiased!) before Nature. This primordial fear of the unknown must be overcome—at least it should be soothed by the Socratic stance that, *anyway, the only thing that we know for sure is that we know almost nothing*—and a way of achieving this is by engaging into imaginative, creative *poetic activity* where there is plenty of leeway for ‘trial-and-error’ and a lot of room for iconoclastic, unconventional and adventurous ideas that are unburdened by ancestral theoretical demands or traditional conventions.

Indeed, granted that QG pushes us back to theorizing about the archeogon acts of the World, what better means other than poetry (with its analogies, metaphors and allegories) do we possess for exploring, conceptually afresh and without a priori commitments—ultimately, to deconstruct and reconstruct anew [35]—the strange,³¹ uncharted QG landscape? Kandinsky’s words echo ecophantically here [11]:

“*Poetry brings us closer to the Creator.*”

Especially regarding the unfamiliar realm of the quantum, we read from [33] (reading from [49]):

“... In the first forty years of the twentieth century, our vision of the physical world changed radically and irretrievably. Atoms could behave like solid matter or like waves, they were made of particles with strange top-like properties, with nuclei which could disintegrate spontaneously, and, perhaps, set up chains of disintegration themselves. For many, the most interesting implication of all this new knowledge was, and still is, philosophical. We have understood that our intuitive ideas of what is possible and what is not—our common sense—are a result of the conditioning of our minds by sense-experiences. *We have had to change our ideas of what understanding consists in.*³² As Bohr said, ‘*When it comes to atoms, language can only be used as in poetry. The poet,*

³⁰Especially the nowadays academic ‘Philosophy of Science’ [2, 45], which appears to be heavily (almost paralytically!) dependent on the concepts, techniques, results and current developments in science (and in particular, in theoretical physics and applied mathematics).

³¹‘Strange’, of course, relative to what we already (think we) know!

³²Midgley’s emphasis.

*too, is not nearly so concerned with describing facts as with creating images.*³³ The same is true of cosmological models, curved spaces and exploding universe. *Images and analogies are the keys.*³⁴ Not you, not I, not Einstein could interpret the universe in terms wholly related to our senses. Not that it is incomprehensible, no. *But we must learn to ignore our preconceptions concerning space, time and matter, abandon the use of everyday language and resort to metaphor. We must try to think like poets. . .*³⁵

What we have in mind here is that, in order to see and tackle the problem of QG afresh, we must foremost be able to sort of ‘(re)create it from scratch’, forgetting for a while the voluminous body of work—the various theoretical ‘evidence’ that different approaches to QG provide us with—that has been gathered over the last 70+ years of research on it. The spirit of Feynman comes to mind:

“*What I cannot create, I do not understand.*” [8]³⁶

Of course, by ‘poetry’ above all we mean *creation of new conceptual terminology within a novel theoretical and technical framework*. In this respect our ADG-inspired QG research endeavors are akin (in spirit at least) to how Feynman wished to tackle the problem of QG. As Hatfield accounts in the prologue to [8]:

“... Thus it is no surprise that Feynman would recreate general relativity from a non-geometrical viewpoint. The practical side of this approach is that one does not have to learn some ‘fancy-schmanzy’ (as he liked to call it) differential geometry in order to study gravitational physics. (Instead, one would just have to learn some quantum field theory.) However, when the ultimate goal is to quantize gravity, Feynman felt that the geometrical interpretation just stood in the way.³⁷ From the field theoretic viewpoint, one could avoid actually defining—up front—the physical meaning of quantum geometry, fluctuating topology, space-time foam, etc., and instead look for the geometrical meaning after quantization... Feynman certainly felt that the geometrical interpretation is marvellous, *‘but the fact that a massless spin-2 field can be interpreted as a metric was simply a coincidence that might be understood as representing some kind of gauge invariance’*³⁸...”

Similarly, within the novel theoretical framework of ADG, ADG-gravity recreates GR purely gauge-theoretically; albeit, unlike Feynman, not by doing away with differential geometry altogether, but by (re)developing and casting the latter in an entirely algebraic and background (spacetime manifold) independent fashion, an achievement that suits perfectly current QG research trends.

In this respect, it is perhaps more important to stress that ADG is not so much a *new* theory of DG—the main *‘mathematical formalism that forms the existing basis of theoretical physics’*, following Dirac’s expression earlier—but a theoretical framework that abstracts,

³³Our emphasis.

³⁴Midgley’s emphasis, and mine.

³⁵Emphasis (and underlining) is all ours.

³⁶In the ‘*Quantum Gravity*’ prologue by Brian Hatfield.

³⁷Recall in this respect from [7]: “*the simple ideas of geometry, extended down to infinitely small, are wrong*”; and more recently Isham [10]: “... *at the Planck-length scale, differential geometry is simply incompatible with quantum theory... [so that] one will not be able to use differential geometry in the true quantum-gravity theory*”.

³⁸Our emphasis of Feynman’s words as quoted by Hatfield.

generalizes, revises and recasts the existing CDG by isolating and capitalizing on its fundamental, *essentially algebraic* (:‘relational’, in a Leibnizian sense) features, which are not dependent at all on a background locally Euclidean geometrical ‘space(time)’ (:manifold). In a way, from the novel viewpoint of ADG, we see ‘old’ and ‘stale’ problems (e.g., the C^∞ -singularities of the manifold and CDG based GR) with ‘new’ and ‘fresh’ eyes. Schopenhauer’s words from [44] immediately spring to mind:

“... Thus, the task is not so much to see what no one has yet seen, but to think what nobody yet has thought about that which everybody sees³⁹...”

On the ‘Idiosyncratic’ Terminology Side The novel perspective on gravity that ADG enables us to entertain is inevitably accompanied by *new terminology*. We have thus not refrained from engaging into vigorous poetic, ‘*lexiplastic*’ activity, so that the dodecalogue above abounds with new, ‘idiosyncratic’ terms for novel concepts hitherto not encountered in the standard theoretical physics’ jargon and literature, such as ‘*gauge theory of the third kind*’, ‘*third quantization*’, ‘*synvariance*’ and ‘*autodynamics*’, to name a few.

In this respect, we align ourselves with Wallace Stevens’ words in [47]:

“... Progress in any aspect is a movement through changes in terminology...”

with the ‘changes in terminology’ in our case being not just superficial (:formal) ‘nominal’ ones introduced as it were for ‘flash, effect and decor’, but necessary ones coming from a *significant change in basic theoretical framework for viewing and actually doing DG in QG*: from the usual geometrical manifold based one (CDG), to the background manifoldless and purely algebraic (:sheaf-theoretic) one of ADG.

The Bottom Line is a Verse: a Word for the World ‘*In the beginning was the Word*’, thus the ultimate task for future QG (re)search is to find the right ‘words’ to begin our theory making about the very beginning of the World. For, to quote Bohr (as quoted in [1]):

“... It is wrong to think that the task of physics is to point out how nature is. *Physics concerns what we can say about nature...*”⁴⁰

As Finkelstein notes,⁴¹

“... The fully quantum theory lies somewhere within the theorizing activity of the human race itself, or the subspecies of physicists, regarded as a quantum system. If this is indeed a quantum entity, then the goal of knowing it completely is a Cartesian fantasy, and *at a certain stage in our development we will cease to be law-seekers and become law-makers.*”⁴²

³⁹All emphasis is ours.

⁴⁰Our emphasis. What could baffle the reader here is the following apparent oxymoron: while on the one hand we seem to advocate the aforesaid principle of ADG-field realism (maintaining that the connection field \mathcal{D} exists ‘out there’ independently of us experimenters, measurers/geometers and theoreticians), on the other we endorse Bohr’s dictum above. Again, there’s no paradox here: what *we* can say about Nature (i.e., in this case, about the field \mathcal{D}) is all encoded in the generalized arithmetics \mathbf{A} that *we* choose to represent it (on \mathcal{E}). However, the \mathbf{A} -functoriality of the dynamics secures the independence of the (dynamics of the) field from our generalized measurements (and hence from our geometrical representations, e.g., ‘spacetime’) in \mathbf{A} (and *in extenso* \mathcal{E} , which is locally a power of \mathbf{A}).

⁴¹In an early draft of [9] given to this author back in 1993.

⁴²For more discussion on this theme, see the section in [40], titled ‘*The Saviors of Physical Law*’, emulating Kazantzakis’ “*The Saviors of God*” [12]. Our emphasis.

It is not clear what happens to the concept of a correct theory when we abandon the notion that it is a faithful picture of nature. Presumably, just as the theory is an aspect of our collective life, its truth is an aspect of the quality of our life. . . .”

And what better means other than our Logos—or better, than our imaginative and creative Logos: our poetic and bardic *Mythos*—do we possess for approximating the archegeal Truth about Nature? Moreover, what a humbling thought this is: that in the end we may find out that this truth is the quintessential quality of our ellogous lives. Then, in a Nietzschean sense [34], *we will have become what we already are: Poets true to our Nature!*

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